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THE INCIDENCE OF PROFITS TAXES IN A
NEO-CLASSICAL GROWTH MODEL*

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Recent tax legislation to stimulate growth and investment by means of a reduction in capital taxes (increased depreciation allowances and investment credits) and the labor opposition to such tax decreases has again raised the question of capital tax (or distributional consequences) to center stage.

To help answer this question, I propose to put forward a simple neo-classical growth model which will indicate the long run incidence of capital taxes when the effect of reduced investment and capital-labor ratio brought about by such taxes has had time to reach equilibrium with gross wages below what they otherwise would have been.

It can easily be seen that a decrease in capital taxation will induce increased investment and capital deepening raising the capital-labor ratio and possibly increasing long run net real wages (even though the capital tax reduction involves an increase in wage taxes). This increase in the capital-labor ratio may be the externality to saving to which Marglin [13] often alludes.

Whether labor would desire such a shift from profit to wage taxes is not clear, even when the result is a rise in the long run net wage rate. If labor owned all capital equally and had a discount rate below the present gross rate of profit, which laborers must if they save, labor itself would desire that the wedge or inefficiency brought about by the profits tax be

totally removed. Once the tax was removed, labor would be able to save to the point where their consumption discount rate was equal to the marginal productivity of capital. This would not necessarily bring about the golden rule suggested by Phelps [18] and Solow [23], since worker's consumption discount rates could easily exceed the rate of technical change plus population growth. (It can be seen that this question is a corollary of the corporate tax shifting work of Harberger [7], Musgrave [16], and Krzyzaniak [9], and is also closely related to the social discount rate issue examined by Baumol [1], Diamond [3], Eckstein [4], Feldstein [5], Harberger [8], Marglin [11, 12], and McKean [14].

If labor owns less than all capital they might not desire even a partial lowering of capital taxes which would raise net real wages. Labor would only desire a shift from profit to wage taxes if the present discounted value of the future increase in gross wages that resulted exceeded the present discounted value of increased wage tax payments. Intuitively, it can be seen that labor will not just want to transfer income to capital because capital has a higher savings propensity. If so, labor would just save more themselves. But, labor may desire lower taxes on capital because: lower capital taxes raise GNP and thus raise government expenditures with government a constant proportion of GNP, or lower the general level of taxation to maintain the same absolute level of government expenditures; or labor's discount rate is above the gross rate of return on capital, which it must be if labor does any positive saving.

This paper will consider only two types of tax and transfer instruments; 1) the rate of government spending, and 2) the tax rate on capital from which

the rate of tax on labor can be deduced. If these are the only two types of transfer possible and wealth is not completely and equally owned by labor, then labor will want capital taxes which reduce GNP below the modified "Golden Rule" (marginal product of capital = consumption discount rate of labor). But, it must be remembered that our first question is the measurement of the incidence of capital taxes, not the determination of the inoptimal level.

These problems have not gone unmentioned in the literature. Musgrave [16] was pessimistic about the question of determination of the incidence of capital taxes when he concluded, "No generalization can be made on a priori grounds about changes in factor shares in the process of growth. By the same token, no categorical statement can be made about the distributional effect of budget policies that expand or retard growth," though he states that, "since the marginal propensity to save tends to rise as we move up the income scale, substitution of a progressive for a regressive tax retards growth." Since his comprehensive book, though, work has been done on this problem by Harberger [7], Krzyzaniak [9], Mieszkowski [15] and Sato [20].

Harberger's analysis of the corporate tax is very interesting but incomplete in that it does not incorporate modern general equilibrium growth theory.

Mieszkowski, working along Harberger's lines, gives an interesting treatment of capital tax shifting, though it also fails to take advantage of the insights offered by general equilibrium growth theory. Diamond [2] also analyzes the capital tax question assuming all capital is equally distributed among workers. Sato [22] and Krzyzaniak [9] have done work similar

to the model presented here, using a less general Cobb-Douglas production function.

Hamada [6] analyzes lump sum transfers from capital to labor and vice versa on wages and profits using a general equilibrium. His research is valuable however limited it is for practical policy questions.

Going further, if there is initially no tax on capital (profits), the marginal product of capital would be equal to labor's consumption discount rate, hence it is difficult to see why labor would ever want to make any transfer to capital. Labor would want lump sum transfers from capital even if the real wage is eventually lowered due to labor having a lower marginal propensity to save than capital. In fact, if labor can get lump sum transfers from capital which they can re-invest free of capital taxes, it would seem that they would want the immediate transfer of all non-labor owned capital.

The Model

Let us postulate an economy with two factors, labor and capital, and
two taxes, an income tax on wages (t_w) and an income tax on profits (t_r).
It van be seen that diminishing tax on profits (t_r) and raising the tax on
wages (t_w), while maintaining the government sector constant (as a proportion
of GNP) will most likely raise the net of tax rate of profits. It is
also possible that this adjustment may raise the net of tax real wage rate.

The reason this distributional adjustment may raise wages is that
lowering the tax on profits (t_r) and raising the tax on wages (t_w) has two
effects or incidences on wages. The direct effect is to bring about a lower
net wage by means of the rise in t_w, but the indirect effect is to raise net
profits bringing about a larger capital stock and a higher capital-labor ratio.
The higher capital-labor ratio will result in a higher gross wage. If the
indirect effect of the tax shift outweighs the direct effect, the change
will raise long run net profits and wages.

This will be considered in a neo-classical growth model in which the
capital stock and therefore the capital-labor ratio are determined by a savings
function. Savings decisions are realized and investment adjusts. Amplifying
this statement, we are going to use a full employment growth model in which
the equilibrium path (capital labor ratio) of our economy is stable before and
after changes in the tax structure. We will compare the two economies at the
same point in time after all equilibrating adjustments of tax changes have
worked themselves out.

The model can therefore be used for determining the differential incidence of various tax structures and the amount of long run shifting which occurs.

Assumptions of the Model:

(1) Two factors of production labor, L, and capital, K.

(2) The production function for the economy as a whole is linear homogeneous of degree one and exhibits diminishing returns to factors.

(3) Perfect competition and full employment of both factors at all times.

(4) No foreign trade or international capital movements.

(5) The consumption goods purchased by capitalists and laborers are of the same capital labor intensities so that we do not have to become involved in price index questions.

The Model:

Our production function is as stated $Q = F(K,L)$. National income (Q) is a function of labor and capital.

$$Q = F(K,L) \text{ or } Q/L = f(k) \text{ where } k = K/L \tag{1}$$

The competitive conditions are

$$r = (1-t_r)f'(k) \tag{2}$$

$$w = (1-t_w)[f(k) - kf'(k)] \tag{3}$$

where r and w are the net rates of profits and wages, respectively, and t_r and t_w are the rates of taxation of profits and wages.

Let us first assume that the size of the Federal Budget is held constant as a proportion of the National Income. Thus, the total taxes collected must be equal to tQ, the government budget.

This leads to the equation:

$$tQ = t_w \cdot w_g \cdot L + t_r \cdot r_g \cdot K$$

where w_g and r_g are respectively the gross wage and profit rates.

This can be rewritten as

$$tf(k) = t_w[f(k) - kf'(k)] + t_r f'(k) \cdot k = t_w f(k) + (t_r - t_w)kf'(k)$$

$$(4)$$

or

$$t = t_w[1-\lambda] + t_r \lambda = t_w + (t_r - t_w)\lambda \text{ where } \lambda = \frac{kf'(k)}{f(k)} \text{ capital's share.}$$

Labor is assumed to be supplied inelastically and to grow logrithmically at the rate n. The supply of labor at any given time is

$$L = L_0 e^{nT} \qquad T = \text{time}$$

Assuming the desired savings of the community is always realized, we get:

$$K' = I_T = S_r \cdot r \cdot K + S_w \cdot w \cdot L + S_g tQ = S, \ K' = \frac{dK}{dT} \qquad (5)$$

or

$$\hat{K} = S_r \cdot r + S_w \cdot wL/K + S_g t \ Q/K, \ \hat{K} = \frac{K'}{K}$$

where S_r, S_w, and S_g are the propensities to save out of profits, wages and government revenues respectively, herein assumed to be constant. Capitalists and laborers are assumed to have the same marginal propensity to save out of profits.

On the full employment steady-state equilibrium growth path, the rate of change of the capital stock will equal the rate of change of the labor supply with a fixed capital-labor ratio.

$$\hat{K} = \hat{L}(=n) \text{ with equilibrium } k=\bar{k} \qquad (6)$$

Since we want to examine the effects of lowering t_r and raising t_w keeping the federal budget constant as a proportion of national output, while (t), t_1 will be treated as a parameter of the system. We will examine the effect of varying t_r on k, r, w, t_w and Q.

Our economy will thus move from one stable equilibrium growth path with k constant and growth steady (at the rate n) to another in which k will presumably be different while n will be the same.

From (5) and (6), we obtain our equation for equilibrium growth:

$$n = \hat{k} = S_r(1-t_r)f'(k) + S_w(1-t_w)[f(k) - kf'(k)]1/k + S_g tf(k)1/k$$

or

$$nk = kS_r(1-t_r)f'(k) + S_w(1-t_w)[f(k)-kf'(k)] + S_g tf(k) \qquad (7)$$

Differentiating (2), (3), (4) and (7) and solving for the effects of varying t_r on k, w, and r $(\dfrac{dk}{dt_r}, \dfrac{dw}{dt_r}$ and $\dfrac{dr}{dt_r})$ we obtain:

$$\frac{dk}{dt_r} = f'(k)(S_w-S_r) \left/ \begin{array}{l} -S_r(1-t_r)f''(k)+S_g/kt[f(k)/k-f'(k)] + S_w/k\{(1-t_w) \\ [f(k)-kf'(k)](1/k)-kf''(k)+tf'(k)-t_r[kf''(k)+f'(k)]\} \end{array} \right. \qquad (8)$$

$$\frac{dr}{dt_r} = \frac{-f'(k)/k \left\{S_w(1-t_w)+S_g t \quad f(k)/k-f'(k)+S_w(t-t_r)f'(k)\right\}}{-S_r(1-t_r)f''(k) \text{ as above}} \qquad (9)$$

$$\frac{dw}{dt_r} = \frac{f'(k)/k \left\{S_r(t-t_r)kf'(k)+[S_g t+S_w(1-t_w)][f(k)-kf'(k)]\right\}}{-S_r(1-t_r)f''(k) \text{ as above}} \qquad (10)$$

We can now calculate the effect of raising the tax on capital (t_r) with the gross tax level t kept constant, on both wage and rents (w and r), but we must first discover the signs of the denominator of these three terms. In appendix I, we prove that the denominators are guaranteed positive by the assumption of stability. Thus stability is a

sufficient (but as we can show not a necessary) condition for a positive denominator.

We can now evaluate the effects of a change in t_r on k, w and r <u>via</u> the numerators of our equations (8), (9) and (10).

Letting the denominators be denoted by D, equation (8) (which tells us the effect of a change in t_r on k) is

$$\frac{dk}{dt_r} = \frac{f'(k)(S_w - S_r)}{D}$$

thus $\quad \frac{dk}{dt_r} < 0$ if $S_w < S_r$

The equilibrium capital-labor ratio (k) will be raised by lowering t_r (and raising t_w to compensate for it) whenever the propensity to save out of profits is greater than the propensity to save out of wages (as we would expect).

Examining equation (9), which tells us the effect of a change in t_r on the rate of return to capital r, and using (4) we discover that $\frac{dr}{dt_r} < 0$ if $1 + \frac{S_g t}{S_w} > t$ when $S_w > 0$. Thus lowering the tax on profits always raises the net rate of profit as long as $t < 1 + \frac{S_g t}{S_w}$, which is highly likely.

Lastly, we will analyze equation (10). As we remember, this equation tells us what effect changing t_r will have on the wage rate net of tax (w). From (10), we find that $\frac{dw}{dt_r} < 0$ if $t_r > t + \left[\frac{S_w}{S_r}(1 - t_w) + \frac{S_g}{S_r} t \right] (\lambda^{-1} - 1)$ where λ is capital's gross share of GNP, $(\frac{kf'(k)}{f(k)})$.

Therefore lowering t_r will raise the wage rate net of tax (t_w) if and only if t_r is higher than the average rate of taxation (t) by

$$\left[\frac{S_w}{S_r}(1-t_w) + \frac{S_g}{S_r} t\right](\lambda^{-1}-1).$$

In Summary:

In order that $\frac{dk}{dt_r}$, $\frac{dr}{dt_r}$ and $\frac{dw}{dt_r}$ all be negative, we need have:

(i) $S_r > S_w$.

(ii) $1 + \frac{S_g t}{S_w} > t$ (if $S_w > 0$, otherwise $S_w(1-t_w) + S_g t > 0$)

(iii) $t_r \geq t + \left[\frac{S_w}{S_r}(1-t_w) + \frac{S_g}{S_r} t\right] (\lambda^{-1}-1)^1$

If (i) is satisfied (the m.p.s. of profit earners is higher than that of wage earners), lowering t_r will raise the equilibrium capital-output ratio and raise the absolute level of the economy's growth path, leaving the long term rate of growth unchanged.[2]

Examining condition (ii), we see that lowering the tax on profits (t_r) will raise the net profit rate (r) in all cases.

We are lastly left with condition (iii) governing the sign of $\frac{dw}{dt_r}$ which in practice will be the only critical condition. If in addition to conditions (i) and (ii) being satisfied condition (iii) is also satisfied, the result of lowering t_r will be to raise r, w, k and F(K,L).[3]

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[1]This rule can alternatively be written as $t_r - t_w \geq \left[\frac{S_w(1-t_w)+S_y t_w}{S_r - S_g}\right]\lambda^{-1}$ by the use of (4) and substitution.

[2]If there is Harrod-neutral labor augmenting technical progress the model will be exactly the same except for the fact that the rate of growth will be n+A instead of n (where A is the rate of technical progress).

[3]For a discussion of the question of excess capital deepening, see appendix II.

Let us substitute reasonable parameters in our alternative rule (iii),
footnote 1, and see how restrictive this condition would be. Assuming $S_r = \frac{1}{2}$,
$S_g = S_w = \frac{1}{20}$, $t_w = \frac{1}{10}$, and $\lambda = \frac{1}{3}$, (iii) becomes

$$t_r \geq t_w + \left[\frac{S_w(1-t_w) + S_g t_w}{S_r - S_g} \right] \lambda^{-1}$$

$$t_r \geq .43$$

A 43% tax on capital would maximize the long run path of real wages in our
model.

Our model so far assumes the rate of saving out of profits is independent
of the rate of return to savings. It also assumes that all government ex-
penditures, except for government savings, benefits no one--neither capital
nor labor. Government expenditures provide no benefits, except saving, but
instead meet costs à la defense, foreign aid, de-pollution, etc., and further
assumes that government revenues and expenditure are a constant proportion
of GNP. These assumptions are the most pessimistic we could make for the
reduction of taxes on capital to increase long run net real wages.

As long as labor has a positive discount rate, we shall never see labor
desiring to have t_r (or $t_r - t_w$) equal to the right hand side of the two
expressions for condition (iii), unless, of course, labor was to own all
capital equally shared, in which case $t_r = 0$ would be labor's desire. This
is true because labor must sacrifice income for increased taxes early in order
to supply the capital for higher wages later.

We have so far assumed that government saves part of its funds and
that the remainder of its revenues are given away in foreign aid or used up

in defense expenditures, anti-pollution expenditures, or other costs of
the economy equal to $t(1-S_g)F$. This is the least optimistic case for both
labor and capital to the extent that government expenditures are GNP in-
creasing benefits for neither labor nor capital.

If government expenditures raised either real wages or profits, they
would benefit either wage earners directly or profit earners directly. They
could also benefit wage earners indirectly if government benefits accruing
to capitalists increase the rate of saving out of capital or the real rate
of return on capital. In case government expenditures only increase real
wages, the increased savings by labor do not benefit capitalists, except
perhaps by raising the GNP and lowering the level of taxation in general
(these effects, though, would probably be small), since increased savings
by labor probably lowers the net rate of return to capital more than they
reduce taxes on capital by expanding GNP. In any case, government expenditures
which benefit anyone will lower the minimum rate of t_r at which net real wages
increase when t_r decreases (rule (iii)).

Let us quickly examine the most optimistic case for labor (who want
to maximize net real wages) where all government expenditures benefit labor.
Here all G is expended on social non-defense, etc., services.
Wages now become:

$$w = (1-t_w)[f(k) - kf'(k)] + tf(k) \tag{11}$$

Differentiating, we obtain:

$$\frac{dw}{dt_r} = (10) + tf'(k)\frac{dk}{dt_r} \tag{12}$$

Solving for the range of t_r in which $\frac{dw}{dt_r} < 0$, we obtain the new condition
(iiia):

$$t_r \geq \frac{1}{S_r} \left\{ S_w t + [S_g t + S_w(1-t_w)](\lambda^{-1}-1) \right\} \tag{13}$$

or, with $S_g = S_w$, and substituting from (4),

$$t_r \geq \frac{S_w(\lambda^{-1}-1)}{S_r - S_w} \quad , \tag{14}$$

a condition which requires lower levels of t_r for $\frac{dw}{dt_r} \leq 0$, than under our

previous assumption of no benefits from government expenditures. This con-

dition is independent of t_w because all t_w is returned to labor. Assuming

$\lambda = \frac{1}{3}$, $S_w = \frac{1}{20}$, and $S_r = \frac{1}{10}$ gives $t_r \geq \frac{2}{9}$. If t_r is below $\frac{2}{9}$, raising the

t_r would increase the long run path of real wages. Let us now set $S_w = S_g = 0$

and examine our previous conditions in the no government benefits case which

(i), (ii) and (iii) become (ia)-(iiia).

(i) $S_r > 0$ (ia) $S_r > 0$

(ii) with $S_g = 0$ (iia) same

       $1 > t$

       with $S_g = S_w = 0$

       $\frac{d_r}{dt_r} < 0$ always

(iii) $t_r \geq t$ (iiia) $t_r \geq 0$

In the case where $S_g = S_w = 0$, and government expenditures are a

constant proportion of GNP which yield no welfare to society, we see that when

$t_r = t_w = t$, any rise in k brought about by lowering t_r raises r, raises k,

and thus r·k and tf, but lowers w because the increase in gross w brought

about by the elevated k is more than taken away by the increase in t_w needed

As long as government revenue and expenditures benefit no one and

$S_w = S_g = 0$, $\frac{dw}{dt_r} < 0$ as long as $t_r \geq t$, but if government revenues are all

spent to the benefit of labor, the rule is $\frac{dw}{dt_r} \leq 0$ when $t_r \geq 0$, as would be

expected.

to maintain G as a constant proportion of GNP.

If we now consider a world in which $S_g = S_w = 0$ and $G = \overline{G}$

$$\frac{dw}{dt_r} = \frac{t_r[f'(k)]^2}{(1-t_r)f''(k)} < 0 \quad \text{when } 1 > t_r > 0 \ .$$

If instead $S_g > 0$, $S_w > 0$ and $G = \overline{G}$, we obtain $\frac{dw}{dt_r} < 0$

when $t_r \geq \frac{S_w}{S_r}(1-t_w)(\lambda^{-1}-1)$ again assuming $\lambda = \frac{1}{3}$, $t_w = \frac{1}{10}$,

$S_w = S_g = \frac{1}{20}$ and $S_r = \frac{1}{2}$ again gives $t_r \geq \frac{9}{50}$.

With $G = \overline{G}$, it does not matter to whom further benefits of increased GNP

accrue, since they are fixed and have no marginal impact.[4]   It must also be

noted that if $S_w = S_r$, labor will always desire $t_r \geq 1$.

Now, let us go back to our original case in which $G = tF(K,L)$.   Given

the assumption that $S_w > 0$ and $S_r = S_{r_c} = S_{r_w} > 0$ (the propensity to save

out of profits is the same for capitalists and workers) following

Pasinetti [17] and Samuelson and Modigliani [19], it becomes obvious that

eventually all capital will be owned by workers.

Proof: assuming $S_g = 0$

(a) $k_w = S_w(1-t_w)[f(k) - kf'(k)] + [S_r(1-t_r)f'(k) - n]k_w$

(b) $S_r(1-t_r)f'(k) - n = -S_w(1-t_w)\left[\frac{f(k)}{k} - f'(k)\right]k_w$

Thus setting $k_w = 0$, substituting (b) into (a), and solving for k_w, we get $k_w = k$.

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[4]Assuming $\lambda = \frac{1}{3}$, $t_w = \frac{1}{10}$, $S_r = \frac{1}{2}$, $S_w = \frac{1}{10}$ and substituting in

$t_r \geq \frac{S_w}{S_r}(1-t_w)(\lambda^{-1}-1)$ yields $t_r \geq \frac{18}{50}$, so present capital taxes may be set at

the level which maximizes long run net wages showing how sensitive this rule

is to S_w.   At $S_w = \frac{1}{4}$ the rule becomes $t_r \geq \frac{9}{10}$ or with $\lambda = \frac{1}{4}$, $S_w = \frac{1}{6}$ gives

$t_r \geq \frac{9}{10}$.

Hence, our assumptions cause relatively all capital to be owned by workers in the long run. This is close to the case examined by Diamond [3], except that capital need not be distributed equally among labor. In practice, the assumption that the net (of tax) propensity to save out of profits is the same for workers and capitalists, $S_r(1-t_r) = S_w(1-t_w)$, may be reasonable, since $S_w < S_r$ while progressive taxes and inheritance taxes mean that $t_{r_c} > t_{r_w}$.

Once labor owns the entire capital stock, capital taxes, t_r, might be reduced to zero. Would $t_r = 0$, $S_r > S_w > 0$ then result in excess capital deepening, $(f'(k) < n)$? With $t_r = t_w = 0$, the equilibrium capital-labor ratio k is determined by:

$$n = S_r k f'(k) + S_w \left[\frac{f(k)}{k} - f'(k) \right] \quad \text{from (7)}$$

Solving for $f'(k)$, we get:

$$f'(k) = \frac{n - S_w(1-t_w)\frac{f(k)}{k}}{s_r - S_w(1-t_w)} \tag{15}$$

and examining (15) to see when $f'(k) \gtreqless n$, we obtain:

$$S_w \lesseqgtr \frac{n(1-S_r)}{\frac{f(k)}{k} - n} \tag{16}$$

Substituting reasonable values for the parameters of (16), $S_r = \frac{1}{2}$, $n = \frac{1}{20}$, and $\frac{f(k)}{k} = \frac{1}{4}$, we calculate $S_w \leq \frac{1}{8}$ for $f'(k) \geq n$. Hence, excess capital deepening is extremely unlikely.

Appendix I:

An investigation of the sign of D the denominator of equation (8), (9) and (10).

For the system to be stable, $\hat{K}$ must be below n when k is above equilibrium k, and $\hat{K}$ must be below n when k is below equilibrium k. Therefore, stability implies $\dfrac{d\hat{K}}{dK}$ be negative.

$$n = \hat{K} = S_r(1-t_r)f'(k) + S_w(1-t_w)[f(k)-kf'(k)]1/k + S_g tf(k)1/k \qquad (11)$$

Graphing $\hat{K}$ and n against k, we see that for stable equilibrium $\dfrac{d\hat{K}}{dK}$ must be negative.
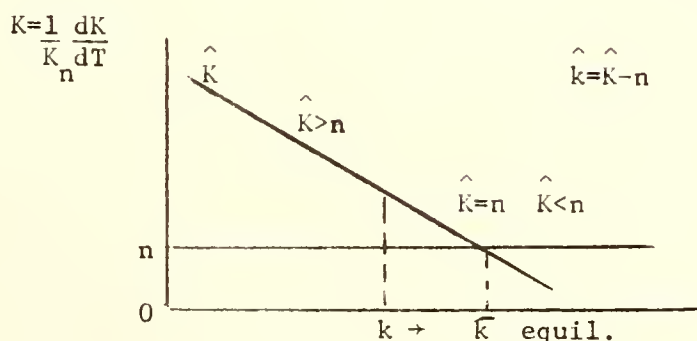
Differentiating $\hat{K}$ with respect to k in (11) and using equation (4) we obtain:

$$\frac{-d\hat{K}}{dK} = -S_r(1-t_r)f''(k) + S_g t[f(k)/k^2 - f'(k)/k] \ +$$

$$S_w/k[kf''(k) + f(k)/k - f'(k) + tf'(k) - tf(k)/k - t_r kf''(k)] \qquad (12)$$

which must be positive for stability.

Now let us examine the denominator of (8), (9), and (10) which we shall denote by capital D. Using equation (4)

$$D = -S_r(1-t_r)f''(k) + S_g t(f(k)/k^2) - f'((k)/k) + \frac{S_w}{k} kf''(k)$$

$$+ f(k)/k - f'(k) + tf'(k) - tf(k)/k - t_r kf''(k)$$

which is equal to $-\dfrac{d\hat{K}}{dk}$ the stability condition which must be positive.

Thus $D = -\dfrac{d\hat{K}}{dk} > 0$ and the denominators of (8), (9) and (10) are all positive.

Appendix II:

An alternative method of looking at our criteria and examining the question of whether or not there is excess capital deepening is to solve (4) and (7) for t_r and t_w yielding:

$$(1-t_r) = \frac{nk-[S_w(1-t) + S_g t]f(k)}{(S_r-S_w)kf'(k)} \qquad (15)$$

$$(1-t_w) = \frac{[S_r(1-t) + S_g t]f(k)-nk}{(S_r-S_w)[f(k)-kf'(k)]} \qquad (16)$$

Our assumption of $t_r, t_r < 1$ implies:

$$[S_w(1-t) + S_g t]f(k) < nk \qquad (17)$$

and:

$$nk < [S_r(1-t) + S_g t]f(k) \qquad (18)$$

$$w = (1-t_w)[f-kf'(k)] = \frac{S_r(1-t) + S_g t]f(k)-nk}{S_r-S_w}$$

We know that in the range where lowering t_r increases $\frac{dk}{dt_r} < 0$ so we can solve for $\frac{dw}{dk} \geq 0$

$$\frac{dw}{dk} = \frac{S_r(1-t) + S_g t]f'(k)-n}{S_r-S_w} \geq 0$$

in any range where

$$\frac{dw}{dk} \cdot \frac{dk}{dt_r} \leq 0$$

$$\frac{dw}{dt_r} \geq 0$$

and

$$[S_r(1-t) + S_g t]f'(k) \geq n$$

$$f'(k) \geq \frac{n}{S_r(1-t) + S_g t}$$

which will almost always be greater than n.

We also see that at maximum w, k will stop increasing before the Golden Rule

is reached, unless t_r is to decrease to the point where w is actually lowered,

bounded by taxes on wages, t_w, being less than one, $t_w \leq 1$.

$$f(k) \geq \frac{nk}{[S_r(1-t) + S_g t]}$$

$$Y \geq \frac{nk}{S_r(1-t) + S_g t} \qquad\qquad \frac{Y}{K} \geq \frac{n}{S_r(1-t) + S_g(t)}$$

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